

Remark on the one-loop Z form factors for LFV Z -penguin diagrams in SUSYE. ARGANDA^{1*}, M.J. HERRERO^{2†}¹*Departamento de Física Teórica, Facultad de Ciencias,
Universidad de Zaragoza, E-50009 Zaragoza, Spain*²*Departamento de Física Teórica and Instituto de Física Teórica, UAM/CSIC
Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain***Abstract**

In this note we shortly comment on our analytical results in PRD73(2006)055003 for the Z boson one-loop form factors contributing to the Lepton Flavour Violating Z penguin diagrams in SUSY. In a recent communication [arXiv:1312.5318v1] it has been pointed out a mistake in our formulas for the chargino contribution to the Z -form factor, $F_L^{(c)}$, and these authors have included in [arXiv:1312.5318v1] corrections to our results which we do not agree with. We wish to clarify here what are the correct results for these form factors and also explain what we believe is the origin of the confusion introduced by the authors in ref.[arXiv:1312.5318v1] about our results. We also wish to single out their incorrect quotations to our work.

*email: ernesto.arganda@unizar.es

†email: maria.herrero@uam.es

In a recent communication [1], the authors are quoting our paper [2] claiming that it contains a mistake in the formulas of the form factors describing the effective Lepton Flavour Violating $l_i - l_j - Z$ vertex that are generated by the one-loop diagrams with MSSM charginos-sneutrinos and neutralinos-sleptons. We wish to clarify in the following our disagreements with their corrections and set the correct formulas for the one-loop Z form factors.

Our convention here for the form factors $F_{L,R}$ defining the effective $Z_\mu l_j l_i$ vertex, the name used for all the functions, couplings, indexes, parameters and particle labels are as in [2]. Thus, the $F_{L,R}$ form factors for the LFV vertex $Z l_j l_i$ are introduced as:

$$-i\gamma_\mu [F_L P_L + F_R P_R] \quad , \quad \text{with} \quad , \quad P_{R,L} = \frac{1}{2}(1 \pm \gamma_5). \quad (1)$$

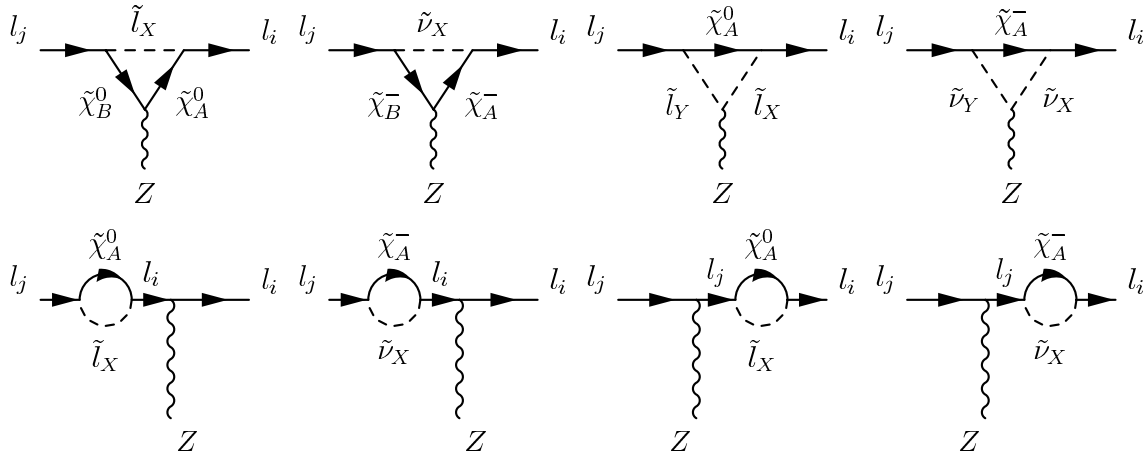


Figure 1: Relevant SUSY one-loop diagrams for the Z -mediated contributions to LFV processes. The particle content is as in the MSSM.

The contributing SUSY one-loop diagrams are those included in [2] that, for clearness, are also collected here in Fig. 1. Notice that we are concerned with the contributing loops in the MSSM context, therefore containing only the sparticles of the MSSM: sneutrinos (SUSY partners of ν_L), sleptons, charginos and neutralinos. We are not concerned here with the possibility of including additional sneutrinos (nor additional neutrinos) beyond the MSSM particle content, like the SUSY partners of the ν_R which have been considered in [1] and also in the recent replica to [1], that just appeared in [3] where they also comment on this same issue.

Our results for the Z -boson form factors in [2] that include the two contributions, from neutralinos (n) and charginos (c), which we repeat here for clearness are:

$$F_{L(R)} = F_{L(R)}^{(n)} + F_{L(R)}^{(c)} \quad , \quad (2)$$

with:

$$F_L^{(n)} = -\frac{1}{16\pi^2} \left\{ N_{iBX}^R N_{jAX}^{R*} \left[2E_{BA}^{R(n)} C_{24}(m_{\tilde{l}_X}^2, m_{\tilde{\chi}_A^0}^2, m_{\tilde{\chi}_B^0}^2) - E_{BA}^{L(n)} m_{\tilde{\chi}_A^0} m_{\tilde{\chi}_B^0} C_0(m_{\tilde{l}_X}^2, m_{\tilde{\chi}_A^0}^2, m_{\tilde{\chi}_B^0}^2) \right] \right. \\ \left. + N_{iAX}^R N_{jAY}^{R*} \left[2Q_{XY}^{\tilde{l}} C_{24}(m_{\tilde{\chi}_A^0}^2, m_{\tilde{l}_X}^2, m_{\tilde{l}_Y}^2) \right] + N_{iAX}^R N_{jAX}^{R*} \left[Z_L^{(l)} B_1(m_{\tilde{\chi}_A^0}^2, m_{\tilde{l}_X}^2) \right] \right\} \quad , \quad (3)$$

$$F_R^{(n)} = F_L^{(n)} \Big|_{L \leftrightarrow R} \quad , \quad (4)$$

$$F_L^{(c)} = -\frac{1}{16\pi^2} \left\{ C_{iBX}^R C_{jAX}^{R*} \left[2E_{BA}^{R(c)} C_{24}(m_{\tilde{\nu}_X}^2, m_{\tilde{\chi}_A^-}^2, m_{\tilde{\chi}_B^-}^2) - E_{BA}^{L(c)} m_{\tilde{\chi}_A^-} m_{\tilde{\chi}_B^-} C_0(m_{\tilde{\nu}_X}^2, m_{\tilde{\chi}_A^-}^2, m_{\tilde{\chi}_B^-}^2) \right] \right. \\ \left. + C_{iAX}^R C_{jAY}^{R*} \left[2Q_{XY}^{\tilde{\nu}} C_{24}(m_{\tilde{\chi}_A^-}^2, m_{\tilde{\nu}_X}^2, m_{\tilde{\nu}_Y}^2) \right] + C_{iAX}^R C_{jAX}^{R*} \left[Z_L^{(l)} B_1(m_{\tilde{\chi}_A^-}^2, m_{\tilde{\nu}_X}^2) \right] \right\}, \quad (5)$$

$$F_R^{(c)} = F_L^{(c)} \Big|_{L \leftrightarrow R}, \quad (6)$$

where the indices are $A, B = 1, \dots, 4$, $X, Y = 1, \dots, 6$ in the contributions from the neutralino sector and $A, B = 1, 2$, $X, Y = 1, 2, 3$ in the contributions from the chargino sector, and a summation over the various indices is understood.

In these formulas we were (as we are now) using a standard notation for the one-loop integrals, B_0 , B_1 , C_0 , C_{24} , based on the original ones in [4]. Specifically, these integrals are defined by:

$$\frac{i}{16\pi^2} B_0(p_1^2, m_1^2, m_2^2) = \int d\tilde{k} \frac{1}{(k^2 - m_1^2)((k + p_1)^2 - m_2^2)} \quad (7)$$

$$\frac{i}{16\pi^2} B_\mu(p_1^2, m_1^2, m_2^2) = \int d\tilde{k} \frac{k_\mu}{(k^2 - m_1^2)((k + p_1)^2 - m_2^2)} \quad (8)$$

$$\frac{i}{16\pi^2} C_0(p_1^2, p_2^2, m_1^2, m_2^2, m_3^2) = \int d\tilde{k} \frac{1}{(k^2 - m_1^2)((k + p_1)^2 - m_2^2)((k + p_1 + p_2)^2 - m_3^2)} \quad (9)$$

$$\frac{i}{16\pi^2} C_{\mu\nu}(p_1^2, p_2^2, m_1^2, m_2^2, m_3^2) = \int d\tilde{k} \frac{k_\mu k_\nu}{(k^2 - m_1^2)((k + p_1)^2 - m_2^2)((k + p_1 + p_2)^2 - m_3^2)} \quad (10)$$

and by:

$$B_\mu = p_{1\mu} B_1 \quad (11)$$

$$C_{\mu\nu} = p_{1\mu} p_{1\nu} C_{21} + p_{2\mu} p_{2\nu} C_{22} + (p_{1\mu} p_{2\nu} + p_{2\mu} p_{1\nu}) C_{23} + g_{\mu\nu} C_{24} \quad (12)$$

where we have used a short notation for the integrals in $D = 4 - \epsilon$ dimension:

$$d\tilde{k} \equiv \mu^{4-D} \frac{d^D k}{(2\pi)^D} \quad (13)$$

These were our analytical results in [2] and we have confirmed now that they are correct. This is contrary to what it is claimed about our result in [1]. Specifically, these authors say the following statement that we wish to point out is incorrect: '...whereas C_{24} is defined by the authors of [5] as $4C_{24}(m_0^2, m_1^2, m_2^2) = B_0(m_1^2, m_2^2) + m_0^2 C_0(m_0^2, m_1^2, m_2^2)$...'. However in our work [5], which is about the different subject 'Lepton flavor violating Higgs boson decays from massive seesaw neutrinos' we did not write this equation, in fact the function C_{24} did not appear at all in this work. Our analytical results in [5] were written, in contrast, in terms of a different function, \tilde{C}_0 which is defined in our works [5] and [2] by:

$$\frac{i}{16\pi^2} \tilde{C}_0(p_1^2, p_2^2, m_1^2, m_2^2, m_3^2) \equiv \int d\tilde{k} \frac{k^2}{(k^2 - m_1^2)((k + p_1)^2 - m_2^2)((k + p_1 + p_2)^2 - m_3^2)}, \quad (14)$$

which is related to B_0 and C_0 by:

$$\tilde{C}_0(p_1^2, p_2^2, m_1^2, m_2^2, m_3^2) = B_0(p_2^2, m_2^2, m_3^2) + m_1^2 C_0(p_1^2, p_2^2, m_1^2, m_2^2, m_3^2), \quad (15)$$

and whose finite part at zero external momenta was given in [2] as:

$$\tilde{C}_0(m_1^2, m_2^2, m_3^2) = 1 - \frac{1}{m_2^2 - m_3^2} \left(\frac{m_1^4 \log m_1^2 - m_2^4 \log m_2^2}{m_1^2 - m_2^2} - \frac{m_1^4 \log m_1^2 - m_3^4 \log m_3^2}{m_1^2 - m_3^2} \right) \quad (16)$$

We want to emphasize that our results in [5] are correct. We would also like to remark that we did not mention at all in any of these two works the alternative function C_{00} , contrary to what is transmitted in [1] and in [3]. Therefore, their comments on our use of C_{00} are misleading.

Finally, and more importantly, we would like to set here the final correct result and specify clearly the only mistake that we admit was done by us in ref. [2], which was exclusively in the formula (B10) of the Appendix B in [2]. We wrote incorrectly the finite part of C_{24} in terms of the finite part of \tilde{C}_0 . Thus, our equation (B10) in [2], where it was incorrectly set $C_{24} = \frac{1}{4}\tilde{C}_0$, should be replaced by this now corrected-(B10):

$$C_{24}(m_1^2, m_2^2, m_3^2) = \frac{1}{4}\tilde{C}_0(m_1^2, m_2^2, m_3^2) + \frac{1}{8} \quad (17)$$

We have redone all the plots in [2] with this corrected-(B10) and we have found no difference at all in any of the numerical results and plots of this article. Thus, this extra $\frac{1}{8}$ is numerically totally irrelevant for our work in [2]. We have also checked that this correction does not change any of our numerical results in refs. [6] and [7] where the Z -penguin diagrams were also participating. Finally, as an extra check of our formulas for the one-loop Z -form factors, we have also found agreement with the corresponding LFV $Zl_i l_j$ effective vertices of refs. [8] and [9] that were computed for other observables.

Acknowledgements

We wish to thank warmly Xabier Marciano, for helping us with all this check. E. A. is financially supported by the Spanish DGIID-DGA grant 2013-E24/2 and the Spanish MICINN grants FPA2012-35453 and CPAN-CSD2007-00042. The work of M. J. H. is partially supported by the European Union FP7 ITN INVISIBLES (Marie Curie Actions, PITN- GA-2011- 289442), by the CI-CYT through the project FPA2012-31880, by the CM (Comunidad Autonoma de Madrid) through the project HEPHACOS S2009/ESP-1473, by the Spanish Consolider-Ingenio 2010 Programme CPAN (CSD2007-00042) and by the Spanish MINECO's "Centro de Excelencia Severo Ochoa" Programme under grant SEV-2012-0249.

References

- [1] M. E. Krauss, W. Porod, F. Staub, A. Abada, A. Vicente and Cd. Weiland, arXiv:1312.5318v1 [hep-ph].
- [2] E. Arganda and M. J. Herrero, Phys. Rev. D **73** (2006) 055003 [hep-ph/0510405].
- [3] A. Ilakovac, A. Pilaftsis and L. Popov, arXiv:1403.3793v1 [hep-ph].
- [4] G. Passarino and M. J. G. Veltman, Nucl. Phys. B **160** (1979) 151.
- [5] E. Arganda, A. M. Curiel, M. J. Herrero and D. Temes, Phys. Rev. D **71** (2005) 035011 [hep-ph/0407302].
- [6] E. Arganda, M. J. Herrero and A. M. Teixeira, JHEP **0710** (2007) 104 [arXiv:0707.2955 [hep-ph]].
- [7] E. Arganda, M. J. Herrero and J. Portoles, JHEP **0806** (2008) 079 [arXiv:0803.2039 [hep-ph]].
- [8] J. I. Illana and M. Masip, Phys. Rev. D **67** (2003) 035004 [hep-ph/0207328].

- [9] T. Fukuyama, A. Ilakovac and T. Kikuchi, Eur. Phys. J. C **56** (2008) 125 [hep-ph/0506295].